

Hong Kong Mathematics Olympiad (2004 – 2005)

Final Event 1 (Group)

香港數學競賽 (2004 – 2005)

決賽項目 1 (團體)

除非特別聲明，答案須用數字表達，並化至最簡。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 一個動物園內有  $a$  頭駱駝，單峯的比雙峯的多 10 頭。若牠們共有 55 個峯，求  $a$  的值。

There are  $a$  camels in a zoo. The number of one-hump camels exceeds that of two-hump camels by 10. If there have 55 humps altogether, find the value of  $a$ .

2. 若  $\text{LCM}(a, b) = 280$  及  $\text{HCF}(a, b) = 10$ ，求  $b$  的值。

If  $\text{LCM}(a, b) = 280$  and  $\text{HCF}(a, b) = 10$ , find the value of  $b$ .

3. 設  $C$  是一正整數且小於  $\sqrt{b}$ 。若  $b$  除以  $C$ ，餘數是 2。除以  $(C+2)$ ，餘數是  $C$ ，求  $C$  的值。

Let  $C$  be a positive integer less than  $\sqrt{b}$ . If  $b$  is divided by  $C$ , the remainder is 2; when divided by  $C+2$ , the remainder is  $C$ , find the value of  $C$ .

4. 一個正  $2C$  邊形共有  $d$  條對角線，求  $d$  的值。

(註：對角線是連接兩個不在同一邊上的頂點的直線。)

A regular  $2C$ -sided polygon has  $d$  diagonals, find the value of  $d$ .

(NB: a diagonal is a straight line joining two vertices not on the same side.)

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Final Event 2 (Group)

香港數學競賽 (2004 – 2005)

決賽項目 2 (團體)

除非特別聲明，答案須用數字表達，並化至最簡。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

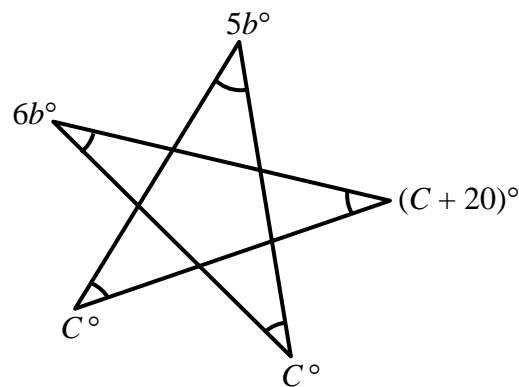
1. 陳先生有 8 個兒子和  $a$  個女兒，他的每個兒子都有 8 個兒子和  $a$  個女兒。他的每個女兒都有  $a$  個兒子和 8 個女兒。已知陳先生的男孫比女孫多 1 個及  $a$  是個質數，求  $a$  的值。

Mr. Chan has 8 sons and  $a$  daughters. Each of his sons has 8 sons and  $a$  daughters. Each of his daughters has  $a$  sons and 8 daughters. It is known that the number of his grand sons is one more than the number of his grand daughters and  $a$  is a prime number, find the value of  $a$ .

2. 設  $\frac{a}{7} = \sqrt[3]{2 + \sqrt{b}} + \sqrt[3]{2 - \sqrt{b}}$ ，求  $b$  的值。

Let  $\frac{a}{7} = \sqrt[3]{2 + \sqrt{b}} + \sqrt[3]{2 - \sqrt{b}}$ . Find the value of  $b$ .

3.



圖一

Figure 1

如圖一，求  $C$  的值。

In Figure 1, find the value of  $C$ .

4. 已知  $P_1, P_2, \dots, P_d$  是  $d$  個連續質數。若  $P_1 + P_2 + \dots + P_{d-2} = P_{d-1} + P_d = C + 1$ ，求  $d$  的值。

Given that  $P_1, P_2, \dots, P_d$  are  $d$  consecutive prime numbers. If  $P_1 + P_2 + \dots + P_{d-2} = P_{d-1} + P_d = C + 1$ , find the value of  $d$ .



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Final Event 3 (Group)

香港數學競賽 (2004 – 2005)

決賽項目 3 (團體)

除非特別聲明，答案須用數字表達，並化至最簡。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 已知  $a$  是方程  $2^{x+1} = 8^{\frac{1}{x} - \frac{1}{3}}$  的正實數解，求  $a$  的值。

Given that  $a$  is a positive real root of the equation  $2^{x+1} = 8^{\frac{1}{x} - \frac{1}{3}}$ . Find the value of  $a$ .

2. 在周界為  $a$  米的長方形中，最大面積的一個長方形的面積是  $b$  平方米，求  $b$  的值。

The area of the largest rectangle with perimeter  $a$  meter is  $b$  square meter, find the value of  $b$ .

3. 若  $c = (1234^3 - 1232 \times (1234^2 + 2472)) \times b$ ，求  $c$  的值。

If  $c = (1234^3 - 1232 \times (1234^2 + 2472)) \times b$ , find the value of  $c$ .

4. 若  $\frac{1}{(c+1)(c+2)} + \frac{1}{(c+2)(c+3)} + \cdots + \frac{1}{(c+d)(c+d+1)} = \frac{8}{15}$ ，求  $d$  的值。

If  $\frac{1}{(c+1)(c+2)} + \frac{1}{(c+2)(c+3)} + \cdots + \frac{1}{(c+d)(c+d+1)} = \frac{8}{15}$ , find the value of  $d$ .

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Final Event 4 (Group)

香港數學競賽 (2004 – 2005)

決賽項目 4 (團體)

除非特別聲明，答案須用數字表達，並化至最簡。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 若  $A^2 + B^2 + C^2 = AB + BC + CA = 3$  及  $a = A^2$ ，求  $a$  的值。

If  $A^2 + B^2 + C^2 = AB + BC + CA = 3$  and  $a = A^2$ , find the value of  $a$ .

2. 已知  $n$  及  $b$  是整數，並滿足方程  $29n + 42b = a$ ，若  $5 < b < 10$ ，求  $b$  的值。

Given that  $n$  and  $b$  are integers satisfying the equation  $29n + 42b = a$ . If  $5 < b < 10$ , find the value  $b$ .

3. 若  $\frac{\sqrt{3} - \sqrt{5} + \sqrt{7}}{\sqrt{3} + \sqrt{5} + \sqrt{7}} = \frac{c\sqrt{21} - 18\sqrt{15} - 2\sqrt{35} + b}{59}$ ，求  $c$  的值。

If  $\frac{\sqrt{3} - \sqrt{5} + \sqrt{7}}{\sqrt{3} + \sqrt{5} + \sqrt{7}} = \frac{c\sqrt{21} - 18\sqrt{15} - 2\sqrt{35} + b}{59}$ , find the value of  $c$ .

4. 若  $c$  有  $d$  個正因數，求  $d$  的值。

If  $c$  has  $d$  positive factors, find the value of  $d$ .